

Excited $L = 1$ baryons in large N_c QCD

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Abstract. The physics of the orbitally excited baryons simplifies drastically in the large N_c limit. The states are arranged into irreducible representations of the contracted $SU(4)_c$ symmetry, with mixing angles determined exactly. The ratios of the strong couplings $N^* \rightarrow [N\pi]_{S,D}$ are predicted in this limit, with results in agreement with those following from the quark model (with the large N_c mixing angles). We present a phenomenological analysis of the observed nonstrange baryons from the perspective of the $1/N_c$ expansion, including constraints from their masses and strong decays.

It has been known for some time that the large N_c limit of QCD [1, 2] can give a useful qualitative description of low energy hadronic physics. In the baryon sector this limit turns out to be considerably more predictive [3, 4, 5], and the $1/N_c$ expansion can be formulated in a systematic way allowing the treatment of power corrections and $SU(3)$ breaking effects [6, 7, 8] (for a recent review see [9]).

The crucial point is the emergence of a new symmetry of QCD in the large N_c limit of the baryon sector - the contracted $SU(2n_f)_c$ symmetry (with n_f the number of light flavors) [6]. The physical states arrange themselves in irreducible representations of this symmetry group, which for $n_f = 2$ are labeled by $K = 0, \frac{1}{2}, 1, \dots$ and contain all states satisfying $|I - J| \leq K$. The leading order predictions for masses and strong couplings recover the quark model with $SU(2n_f)$ spin-flavor symmetry. Using this approach, many applications have been discussed for the ground state baryons [9, 10, 11].

The large N_c expansion has been applied also to excited baryons, using different implementations of the idea [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. In Refs. [14, 15] the contracted $SU(4)_c$ symmetry was found to extend also to these states, using consistency conditions for $N^{(*)}\pi \rightarrow N^{(*)}\pi$ scattering. However, because of the more complex mass spectrum of the excited states, the implications of this symmetry are more rich than in the ground state sector. In particular, the nonstrange negative parity $L = 1$ baryons fall into three irreducible representations of the $SU(4)_c$ symmetry

$$\begin{aligned} K = 0 : & \quad N_{1/2}, \Delta_{3/2}, \dots \\ K = 1 : & \quad N_{1/2}, N_{3/2}, \Delta_{1/2}, \Delta_{3/2}, \Delta_{5/2}, \dots \\ K = 2 : & \quad N_{3/2}, N_{5/2}, \Delta_{1/2}, \Delta_{3/2}, \Delta_{5/2}, \Delta_{7/2}, \dots \end{aligned} \tag{1}$$

This can be contrasted with the case of the corresponding ground state baryons, which include only one representation $K = 0 : N_{1/2}, \Delta_{3/2}, \dots$. Thus, the large N_c limit implies a mass pattern for the excited baryons Eq. (1) which is very different from the quark model prediction of complete degeneracy into the **70** of $SU(6)$ [23, 24]. Still, the large

N_c predictions for $N^* \rightarrow N\pi$ amplitude ratios are found to be again in agreement with those of the quark model with SU(4) spin-flavor symmetry [14]. [We note that very similar predictions are obtained for hybrid baryons in the large N_c limit [25].]

In a recent paper [26], the status of the $1/N_c$ expansion for the nonstrange $L = 1$ baryons was reexamined, working consistently at leading and subleading order in $1/N_c$. This was done using the operator approach proposed in [7, 6], and first applied to the excited states in [13, 16, 17]. The mass matrix of the $L = 1$ baryons can be written as a sum of operators acting on the quark basis as

$$\hat{M} = \sum_{k=0}^{N_c} \frac{1}{N_c^{k-1}} C_k \mathcal{O}_k \quad (2)$$

with \mathcal{O}_k a k -body operator. Both the coefficients C_k and the matrix elements of the operators on baryon states $\langle \mathcal{O}_k \rangle$ have power expansions in $1/N_c$ with coefficients determined by nonperturbative dynamics

$$C_k = \sum_{n=0}^{\infty} \frac{1}{N_c^n} C_k^{(n)}, \quad \langle \mathcal{O}_k \rangle = \sum_{n=0}^{\infty} \frac{1}{N_c^n} \langle \mathcal{O}_k \rangle^{(n)}. \quad (3)$$

The natural size for the coefficients $C_k^{(n)}$ is $\Lambda \sim 500$ MeV. A complete basis for the operators $\mathcal{O}_k^{(1,2)}$ has been constructed in [17], to which we refer for further details. At leading order in N_c only three operators contribute to the mass matrix, given by

$$O_1 = N_c \mathbf{1}, \quad O_2 = l^i s^i, \quad O_3 = \frac{3}{N_c} l^{(2)ij} g^{ia} G_c^{ja}. \quad (4)$$

At subleading order $O(N_c^{-1})$ five additional operators start contributing

$$O_4 = ls + \frac{4}{N_c + 1} lt G_c, \quad O_5 = \frac{1}{N_c} l S_c, \quad O_6 = \frac{1}{N_c} S_c S_c, \quad O_7 = \frac{1}{N_c} s S_c, \quad O_8 = \frac{1}{N_c} l^{(2)} s S_c. \quad (5)$$

Their matrix elements on the excited baryon states can be found in the Appendix of [17]. These operators have a direct physical interpretation in the quark model in terms of one- and two-body quark-quark couplings.

Keeping only the operators $O_{1,2,3}$ contributing at $O(N_c^0)$, one finds by direct diagonalization of the mass matrix the mass eigenstates in the large N_c limit as linear combinations of the quark model $N_{1/2}, N'_{1/2}$ states

$$\begin{aligned} |K=0, J=\frac{1}{2}\rangle &= \frac{1}{\sqrt{3}} N_{1/2} + \sqrt{\frac{2}{3}} N'_{1/2} \\ |K=1, J=\frac{1}{2}\rangle &= -\sqrt{\frac{2}{3}} N_{1/2} + \frac{1}{\sqrt{3}} N'_{1/2} \end{aligned} \quad \begin{cases} M_0^{(0)} = N_c C_1^{(0)} - C_2^{(0)} - \frac{5}{8} C_3^{(0)} \\ M_1^{(0)} = N_c C_1^{(0)} - \frac{1}{2} C_2^{(0)} + \frac{5}{16} C_3^{(0)} \end{cases} \quad (6)$$

A similar diagonalization of the mass matrix for the $J = \frac{3}{2}$ N^* states gives the eigenstates

$$\begin{aligned} |K=1, J=\frac{3}{2}\rangle &= \frac{1}{\sqrt{6}} N_{3/2} + \sqrt{\frac{5}{6}} N'_{3/2} \\ |K=2, J=\frac{3}{2}\rangle &= -\sqrt{\frac{5}{6}} N_{3/2} + \frac{1}{\sqrt{6}} N'_{3/2} \end{aligned} \quad \begin{cases} M_1^{(0)} = N_c C_1^{(0)} - \frac{1}{2} C_2^{(0)} + \frac{5}{16} C_3^{(0)} \\ M_2^{(0)} = N_c C_1^{(0)} + \frac{1}{2} C_2^{(0)} - \frac{1}{16} C_3^{(0)} \end{cases} \quad (7)$$

TABLE 1. The four possible assignments of the observed nonstrange excited baryons into large N_c towers with $K = 0, 1, 2$.

	K = 0	K = 1	K = 2	ordering
# 1	$N_{1/2}(1650)$	$\{N_{1/2}(1535), N_{3/2}(1520)\}$	$\{N_{3/2}(1700), N_{5/2}(1675)\}$	$\{M_0, M_2\} > M_1$
# 2	$N_{1/2}(1535)$	$\{N_{1/2}(1650), N_{3/2}(1520)\}$	$\{N_{3/2}(1700), N_{5/2}(1675)\}$	$M_2 > M_1 > M_0$
# 3	$N_{1/2}(1535)$	$\{N_{1/2}(1650), N_{3/2}(1700)\}$	$\{N_{3/2}(1520), N_{5/2}(1675)\}$	$M_1 > \{M_0, M_2\}$
# 4	$N_{1/2}(1650)$	$\{N_{1/2}(1535), N_{3/2}(1700)\}$	$\{N_{3/2}(1520), N_{5/2}(1675)\}$	$M_0 > M_1 > M_2$

The $N_{5/2}$ state does not mix and has the mass $M_2^{(0)}$. These results make the tower structure in Eq. (1) explicit.

There is a discrete ambiguity in the assignment of the five observed N^* excited nucleons into the large N_c irreducible reps of $SU(4)_c$. The four possible ways of grouping them into multiplets are shown in Table 1. This implies a four-fold ambiguity in the coefficients of the mass operator $C_i^{(0)}$. In the following we extract these coefficients and attempt to resolve the discrete ambiguity by using experimental information on masses and strong decays of these states.

We start by determining the values of the coefficients $C_{1,2,3}^{(0)}$ in the large N_c limit, using the mass eigenvalues given in Eqs. (6) and (7). For each assignment, we fitted the coefficients $C_{1,2,3}^{(0)}$ to the observed N^* masses [26]. The results for $C_{2,3}^{(0)}$ are shown graphically in Figure 1 for each of the four possible assignments. The mixing angles θ_{N1}, θ_{N3} are fixed by Eqs. (6) and (7).

These results must satisfy an additional constraint, following from the no-crossing property of the eigenstates with the same quantum numbers. Consider the masses of the two $J = 1/2$ states as functions of $1/N_c$. They can not cross when N_c is taken from 3 to infinity. This means that the correspondence of the physical $N_{1/2}$ states with the large N_c towers is fixed by the relative ordering of the $K = 0, 1$ towers. This leads to a connection between the ordering of the tower masses and each of the 4 assignments, shown in the last column of Table 1. This constraint is also shown graphically in Fig. 1; it rules out the assignment No.4 and further restricts the solution for the assignment No.2.

We consider next also information from the strong decays $N^* \rightarrow [N\pi]_{S,D}$. The large N_c predictions for these decays were given in [14], where the consistency conditions for $N^{(*)}\pi \rightarrow N^{(*)}\pi$ scattering were solved exactly. Referring to Ref. [14, 15] for the full solution, we list in Eqs. (9) the results for the S- and D-wave reduced amplitudes A_{red} [defined up to spin and isospin CG coefficients as $A(N^* \rightarrow N\pi) = A_{\text{red}} \cdot CG_I \cdot CG_J$]. The 0's in these relations denote $1/N_c$ suppressed amplitudes. Including spin and isospin factors, these relations predict the large N_c partial width ratios shown in Eq. (10). In addition to constraining the masses of the tower states, the contracted $SU(4)_c$ symmetry relates also their strong decay widths which are predicted to be equal. This equality holds also for individual channels, which implies sum rules such as (for the $K = 2$ states)

$$\Gamma(N_{3/2} \rightarrow [N\pi]_D) + \Gamma(N_{3/2} \rightarrow [\Delta\pi]_D) = \Gamma(N_{5/2} \rightarrow [N\pi]_D) + \Gamma(N_{5/2} \rightarrow [\Delta\pi]_D). \quad (8)$$

These relations are broken by $1/N_c$ terms in the expansion of the $N^* \rightarrow N$ axial current, and by kinematical phase space effects.

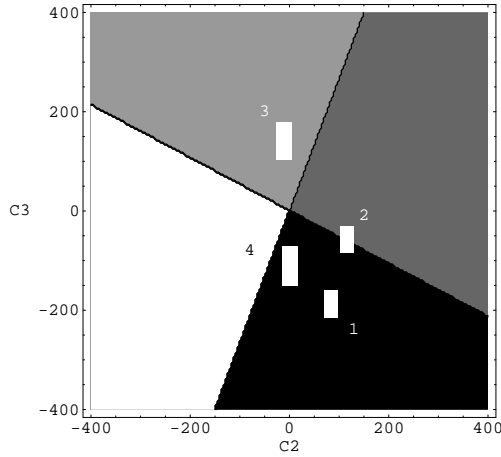


FIGURE 1. Fit results for the leading order coefficients $C_{2,3}^{(0)}$, corresponding to each assignment. The four wedgelike regions show the allowed values for each assignment following from the noncrossing argument.

$K = 0$	$K = 1$	$K = 2$	
$(N_{\frac{1}{2}} \rightarrow [N\pi]_S) = 0$	$(N_{\frac{1}{2}} \rightarrow [N\pi]_S) = \sqrt{2}c_S$ $(N_{\frac{3}{2}} \rightarrow [\Delta\pi]_S) = c_S$	$(N_{\frac{3}{2}} \rightarrow [\Delta\pi]_S) = 0$	
$(N_{\frac{1}{2}} \rightarrow [\Delta\pi]_D) = 0$	$(N_{\frac{1}{2}} \rightarrow [\Delta\pi]_D) = c_{D1}$ $(N_{\frac{3}{2}} \rightarrow [N\pi]_D) = -2c_{D1}$ $(N_{\frac{3}{2}} \rightarrow [\Delta\pi]_D) = -c_{D1}$	$(N_{\frac{3}{2}} \rightarrow [N\pi]_D) = c_{D2}$ $(N_{\frac{3}{2}} \rightarrow [\Delta\pi]_D) = -\frac{1}{2}c_{D2}$ $(N_{\frac{5}{2}} \rightarrow [N\pi]_D) = \sqrt{\frac{2}{3}}c_{D2}$ $(N_{\frac{5}{2}} \rightarrow [\Delta\pi]_D) = \frac{1}{2}\sqrt{\frac{7}{3}}c_{D2}$	(9)

Note that these predictions depend crucially on the K assignment of the excited baryons. In particular, the strong couplings of the $K = 0$ states are suppressed by $1/N_c$. Also, the $J = 3/2$ $K = 2$ state is predicted to decay in a pure D -wave. Therefore one expects these predictions to be useful for distinguishing among the possible assignments.

$$\begin{aligned}
K = 1 : \Gamma(N_{\frac{1}{2}} \rightarrow [N\pi]_S) : \Gamma(N_{\frac{3}{2}} \rightarrow [\Delta\pi]_S) &= 1 : 1 \\
K = 1 : \Gamma(N_{\frac{1}{2}} \rightarrow [\Delta\pi]_D) : \Gamma(N_{\frac{3}{2}} \rightarrow [N\pi]_D) : \Gamma(N_{\frac{3}{2}} \rightarrow [\Delta\pi]_D) &= 2 : 1 : 1 \\
K = 2 : \Gamma(N_{\frac{3}{2}} \rightarrow [N\pi]_D) : \Gamma(N_{\frac{3}{2}} \rightarrow [\Delta\pi]_D) : \Gamma(N_{\frac{5}{2}} \rightarrow [N\pi]_D) : \Gamma(N_{\frac{5}{2}} \rightarrow [\Delta\pi]_D) \\
&= \frac{1}{2} : \frac{1}{2} : \frac{2}{9} : \frac{7}{9}.
\end{aligned} \tag{10}$$

For this purpose, we consider the ratios of S -wave partial widths $R_1 = \frac{\Gamma(N_{1/2}(1535) \rightarrow N\pi)}{\Gamma(N_{3/2}(1520) \rightarrow [\Delta\pi]_S)}$ and $R_2 = \frac{\Gamma(N_{1/2}(1650) \rightarrow N\pi)}{\Gamma(N_{1/2}(1535) \rightarrow N\pi)}$. We present in Table 2 the large N_c

TABLE 2. Large N_c predictions for the ratios of strong decay widths $R_{1,2}$, and their experimental values.

	R_1	R_2	$R_1 R_2$
# 1	2.05	$O(1/N_c^2)$	$O(1/N_c^2)$
# 2	$O(1/N_c^2)$	$O(N_c^2)$	2.4
# 3	$O(1)$	$O(N_c^2)$	$O(N_c^2)$
# 4	$O(N_c^2)$	$O(1/N_c^2)$	$O(1)$
exp	3.6-13.7	1.0-2.6	5.7-22.5

predicted values for $R_{1,2}$ (including phase space factors), together with their experimental values.

Despite the large experimental errors, the combined constraints from masses and strong decays ($R_{1,2}$) appear to favor the assignment No.1 [14, 26]. In Ref. [26] the mass analysis presented here was extended to $O(1/N_c)$. The most important new point of this analysis is the appearance of a continuous set of solutions for the mass operator. Including also data from excited Δ states, it was found that the assignment No.3 is favored, in agreement with the analysis in [17], although No. 1 is still marginally allowed. More conclusive results will be possible once better data on the masses and decay widths of these states will become available.

Finally, we note that similar conclusions on the mass spectrum of these states (1) were also reached in [27, 28, 29] from a study of $N\pi$ scattering amplitudes in the Skyrme model.

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